Predictability and Dynamics of Geophysical Fluid Flows

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LONG-TERM GOALS

The long-term goal of the research in this project is to improve our ability to predict environmental conditions using dynamical models.

OBJECTIVES

The central objective of the research in this project is to understand the mathematical and physical connections between the bred-growing-mode and singular vector techniques recently developed for numerical weather prediction, the Lyapunov vectors and exponents of dynamical systems theory, and instability theories of geophysical fluid dynamics. The intent is to gain insight into fundamental mathematical and physical aspects of predictability in unstable (irregular, chaotic) continuous systems.

APPROACH

A combination of analytical and numerical methods are being used to study a variety of mathematical models of geophysical fluid flows.

WORK COMPLETED

Graduate student Christopher Wolfe, who has been supported by this grant, defended his thesis on this topic in September 2006 (Wolfe, 2006). A new, efficient method for recovering Lyapunov vectors from singular vectors has been discovered and tested (Wolfe and Samelson, 2006b). A singular vector analysis of disturbances to the strongly nonlinear baroclinic oscillation has been completed and is described in a manuscript that will soon be submitted (Wolfe and Samelson, 2006c). Under the previous grant (N00014-98-1-0813), the Floquet analysis of linear disturbances to a strongly nonlinear baroclinic wave-mean oscillation in a high-dimensional geophysical fluid model was carried out (Wolfe and Samelson, 2006a).

RESULTS

Under the previous grant, the stability of a time-periodic baroclinic wave-mean oscillation in a two-layer quasi-geostrophic model was examined by computing a full set of time-dependent normal modes (Floquet vectors) for the oscillation (Wolfe and Samelson, 2006a). The Floquet vectors fell into two classes with direct physical interpretations: wave dynamical (WD) modes and damped-advective (DA) modes. The dynamics of the WD modes reflects the dynamics of the wave-mean oscillation, and these

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modes are analogous to the normal modes of steady parallel flow. The DA modes represent instead a generalized solution to the damped advection problem. The leading Floquet exponents provide useful approximations to the leading Lyapunov exponents of the associated chaotic solution.

A new method for recovering Lyapunov vectors from asymptotic singular vectors has been discovered and tested (Wolfe and Samelson, 2006b). Lyapunov vectors are the fundamental time-dependent modes of instability of a flow with arbitrary time-dependence, and reduce to standard normal modes or Floquet modes, respectively, for steady or time-periodic basic flows. Standard methods require $N + 1$ singular vectors for the computation of any Lyapunov vector except the first, where N is the number of degrees of freedom of the model, which in ocean or atmosphere models is typically of order 105-107. The new method requires only $2n$ singular vectors for the computation of n Lyapunov vectors. Since the number n of leading Lyapunov vectors of interest is typically of order 10-102, this is a substantial savings; from a practical point of view, it makes calculations possible that previously were impossible. This method has been tested successfully on a weakly nonlinear model with 8 degrees of freedom (Figure 1) and on a strongly nonlinear model with 3840 degrees of freedom (Figure 2).

The analysis of the relationship between singular vectors and Floquet vectors in the context of a nonlinear baroclinic wave-mean oscillation was completed (Wolfe and Samelson, 2006c). It was found that the singular vectors divide into two dynamical classes which are related to those of the Floquet vectors. Singular vectors in the WD class were found to asymptotically approach constant linear combinations of Floquet vectors. The most rapidly decaying singular vectors projected strongly onto the most rapid decaying Floquet vectors. In contrast, the leading singular vectors projected strongly onto the leading adjoint Floquet vectors. Examination of trajectories near the basic cycle showed that the leading Floquet vectors were geometrically tangent to the local attractor, and so efficiently represented error variance associated with misplacement of the state on the attractor (Figure 3), while the leading initial singular vectors pointed off the local attractor, and so represented this error variance inefficiently (Figure 4). This illustrates that, for the standard linear and perfect-model hypotheses, the time-dependent normal modes (Lyapunov vectors) are a more appropriate basis for ensemble generation than singular vectors.

IMPACT/APPLICATIONS

The primary potential future impact of these results is on the design and use of ensemble forecasting techniques for the prediction of oceanic and atmospheric conditions.

RELATED PROJECTS

This project is a continuation of research that formed part of the ONR Predictability DRI. The research on dynamics and predictability is related to the NSF ITR project "Spatio-Temporal Complexity and Nonlinear Dynamics of Coastal Ocean Flows" (R. Samelson, J. Allen, and G. Egbert, Co-PIs).
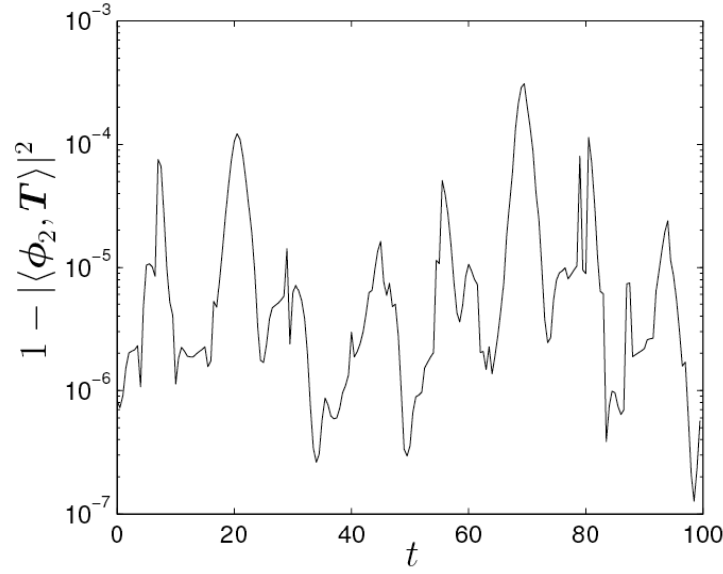
Figure 1: One minus the pattern correlation between the second Lyapunov vector and the tangent vector to the nonlinear trajectory of the weakly nonlinear Phillips model vs. time t along the basic cycle. A value of unity indicates orthogonality while a value of zero indicates collinearity.
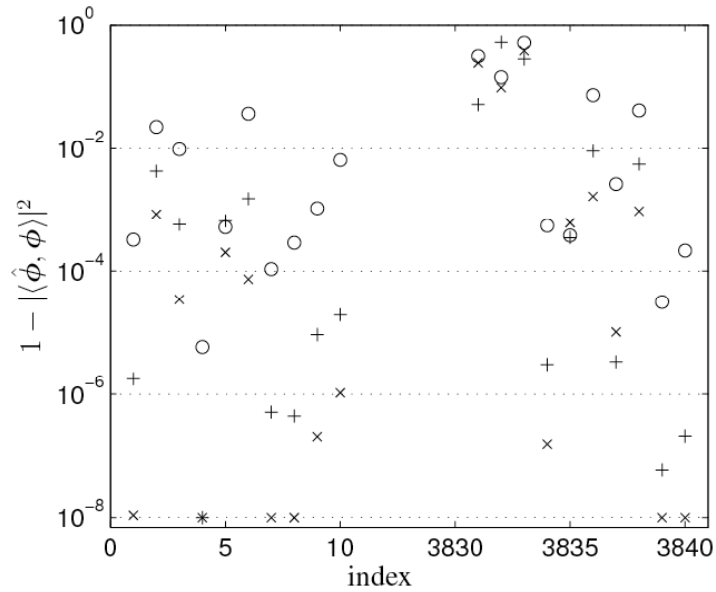
Figure 2. One minus the pattern correlation between the recovered Floquet vectors and the true Floquet vectors for three different optimization intervals (crosses, pluses, circles) vs. mode number. A value of unity indicates orthogonality while a value of zero indicates collinearity. Pattern correlations < 10-8 are plotted at 10-8 to show the upper values more clearly.
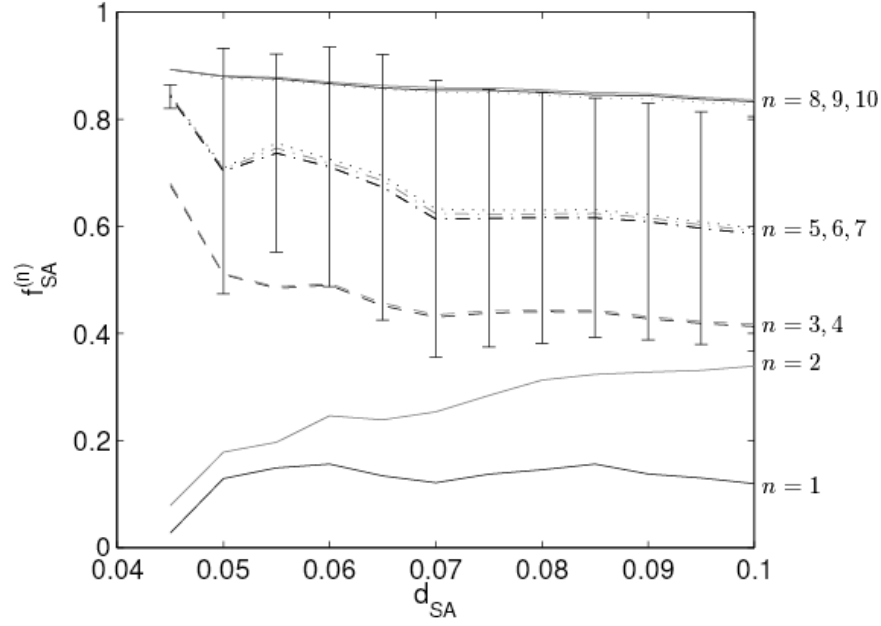
Figure 3. Fraction of the initial error variance explained by the leading Floquet vectors (vertical axis) vs. maximum error bound. Lines give bin averages and errorbars (plotted on the n = 5 line) give bin standard deviations. The bin size is 0:005. The number of Floquet vectors n used to construct the leading subspace increases vertically, starting with n = 1 and ending with n = 10.
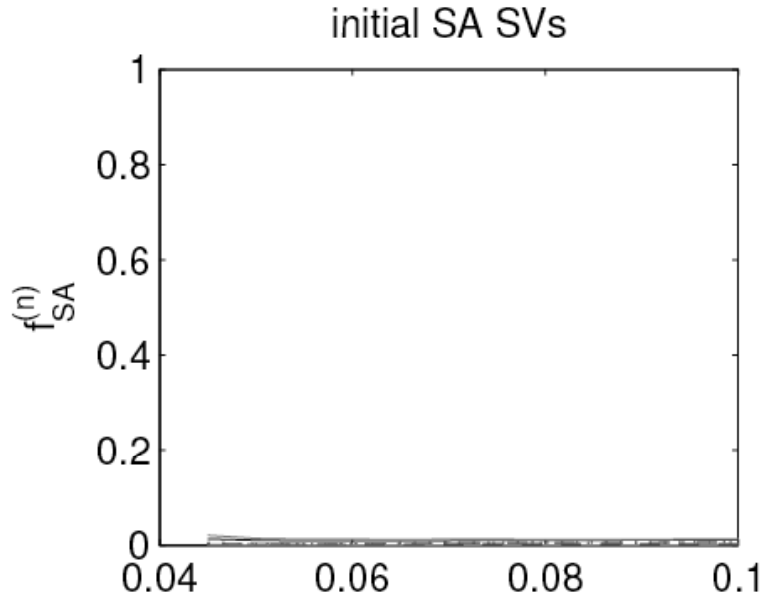
Figure 4. Fraction of the initial error variance explained by the leading singular vectors (vertical axis) vs. maximum error bound. The bin size is 0:005. The number of singular vectors n used to construct the leading subspace increases vertically, starting with n = 1 and ending with n = 10. All values are less than 0.05.

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Wolfe, C. L., and Samelson, R. M., 2006a. Normal mode analysis of a baroclinic wave-mean oscillation. Journal of the Atmospheric Sciences.

PUBLICATIONS

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